DOI: 10.21858/msr.19.10

Voronoi diagrams – rod structure research models in architectural and structural optimization

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ABSTRACT

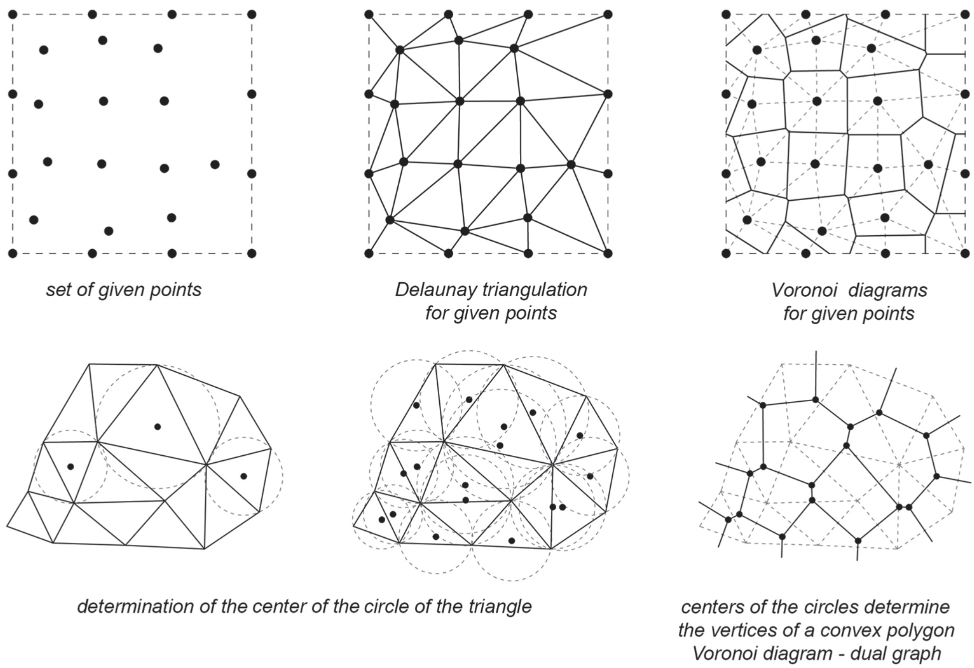
In modern design architecture there is an interesting trend of developing highly complex spatial systems, inspired by patterns inspired by the natural world. As a result, visually innovative solutions are being created, and what is important from an engineering point of view, also technologically efficient. The logic behind shaping "bionic" architecture is aimed at, among others, the rational use of materials and energy. The search for new spatial structures through multi-criteria analyses has increased the interest in digital optimization issues in multidisciplinary architectural and engineering analyses. Algorithms used to form diverse, multi-variant carrier systems based on similar boundary assumptions play a special role in improving computer programs. Research samples in the development of regular and irregular rod-like structures based on own developed digital models have been presented as an example of such action. The simulations conducted involved analysis of configurable structures based on their efficiency in material consumption.

Introduction

The modern tools of the architect constitute an environment of computer programs used by designers at all stages of the design, from the early concept to detailed shop drawings. An important feature of the digitization process is its interdisciplinary design that encapsulates the interactions of various technical fields. Thanks to the digital modeling integrating systems such as BIM (Building Information Modeling) advanced multi-branch cooperation is possible involving the search for interdependent rational solutions, which are often a compromise to the set parameters. In terms of shaping architectural rod structures, the interdisciplinary design leads to sophisticated structural solutions characterized by an unconspicuous, complex shape, the geometry of which referring to free fractal structures, sometimes exceeding the paradigms of Euclidean geometry [Pottmann H., Asperl A., Hofer M., Kilian A., 2007].

Bionic projection plays an important role in the search for new architectural rod structure solutions. As the improvement of research tools continues, so does the knowledge of the surface structure of living organisms, which in turn is used as a source of inspiration in fields such as architecture (particularly in architectural structures), construction and materials science [Burry J., Burry M., 2010]. One of the most interesting bionic discretization methods of structural surfaces is the Dirichlet tessellation, also referred to as the Voronoi Diagram, which describes the division planes and space which is encountered in nature, such as the pattern of a dragonfly wing, the spots on the giraffe's skin, or the tortoiseshell. Currently, the Voronoi diagram is one of the rules in mathematics and geometry inspired by bionics as

Fig. 1. Delaunay Triangulation and Voronoi Diagram – divisions also represent a dual graph relative to each other; a – creating the Voronoi Diagram based on the Delaunay triangular grid divisions; b – Delaunay triangulation and Voronoi diagram for a given group of points



an alternative to regular tessellation known and used in architecture [Gawell E., Rokicki W., Nowak A., 2014]. The characteristic distribution of the two-dimensional Voronoi surface for a given set of *n* points of any plane formed by such division into *n* regions, where each point in the region is closer to a specified point of a set of *n* points than from the other *n-1* points.

The Delaunay triangulation is a dual graph for a Voronoi Diagram, as shown in the figure below (Fig. 1). The Delaunay Triangulation *(T)* means such a division of space *(Rn + 1)* into convex polygons (simplexes), that two *T* simplexes have a common wall or have no common parts; each limited *Rn + 1* set has a shared part only with a finite number *T* simplexes; the inside of a sphere inscribed in any *T* simplex does not contain any vertices of the *T* simplex.

Today, the Delaunay divisions have a more practical application in the design of architectural forms created from rod structures. The solutions for such systems are obtained using a generative model, and the algorithm responsible for digitizing the surface is incorporated and embedded in 3D modeling software. A significant advantage of using digital generators in the search for optimal solutions in architecture and design is the ability to model multi-variant solutions, as well as the ease of modification (the model arises as a result of implemented numerical data iterations).

The roof of the WestendGate building in Frankfurt am Main in Germany is one of interesting examples of such a structured surface. The Just Burgeff Architekten and a3lab (Asterios Agkathidis Architecture) project is an example of the Voronoi Diagram application in the pedestrian passage roofing with arboreal supports (Fig. 2a). The roofing was built in 2010 as one of the important elements of a skyscraper featuring office and hotel functions, known as the Marriott Hotel. The public space around the building has been changed due to the reconstruction of the building following the relocation of the entrance to the underground garage. This resulted in additional space allocation. The proximity of green areas warranted change of the project to transform it into an urban recreation zone. Consequently, it was decided to open the space toward the city and construct roofing, which would form a new gateway to the WestendGate. The roofing project is based on biomorphic shapes describing the algorythm for growth and formation of living organisms. For a rational use of material, which in this case was steel, a structural mesh has been optimized by means of the Finite Element Method. The curved surface of the roof was tested using specialized computer programs, which resulted in a force distribution model of the structure. The crucial use of software facilitated a 3D model generation and its management, in view of the optimization possibilities and the application of modern production methods. The use of arboreal supports, resembling the shape of tree branches, made it possible to reduce the number of supports and the maximum support of a 1000 m² roof surface with a variable height of up to 14 meters.

Fig. 2. Voronoi Diagram Application in discretization of structural surfaces; a – WestendGate shelter in Frankfurt, Just Burgeff Architekten and a3lab project, 2010; Grotto for Meditation in Houston, Metalab Architecture & Fabrication project with scientists from the College of Architecture in Houston; b – generating Voronoi divisions – seeking rational solutions; c – digital end model; d – the finished view of pavilion

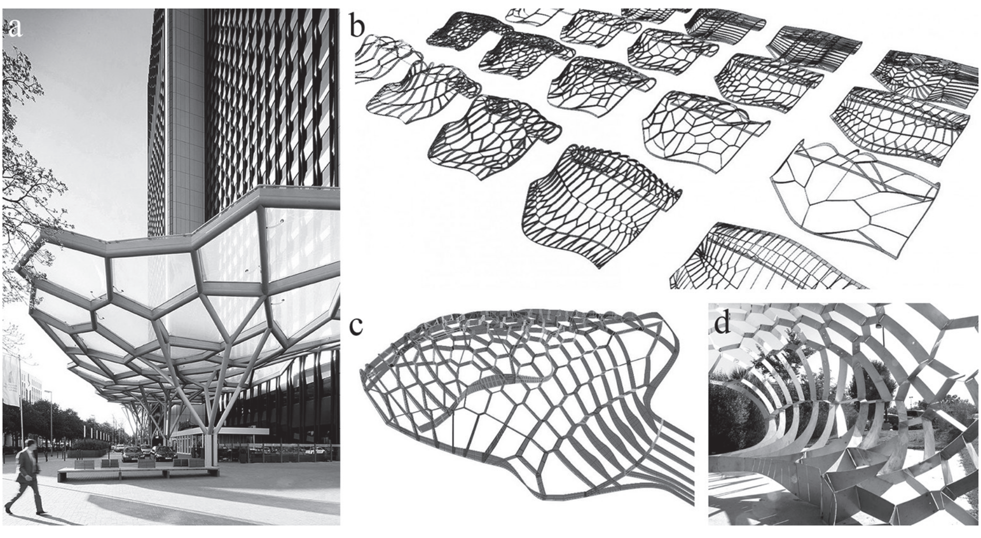
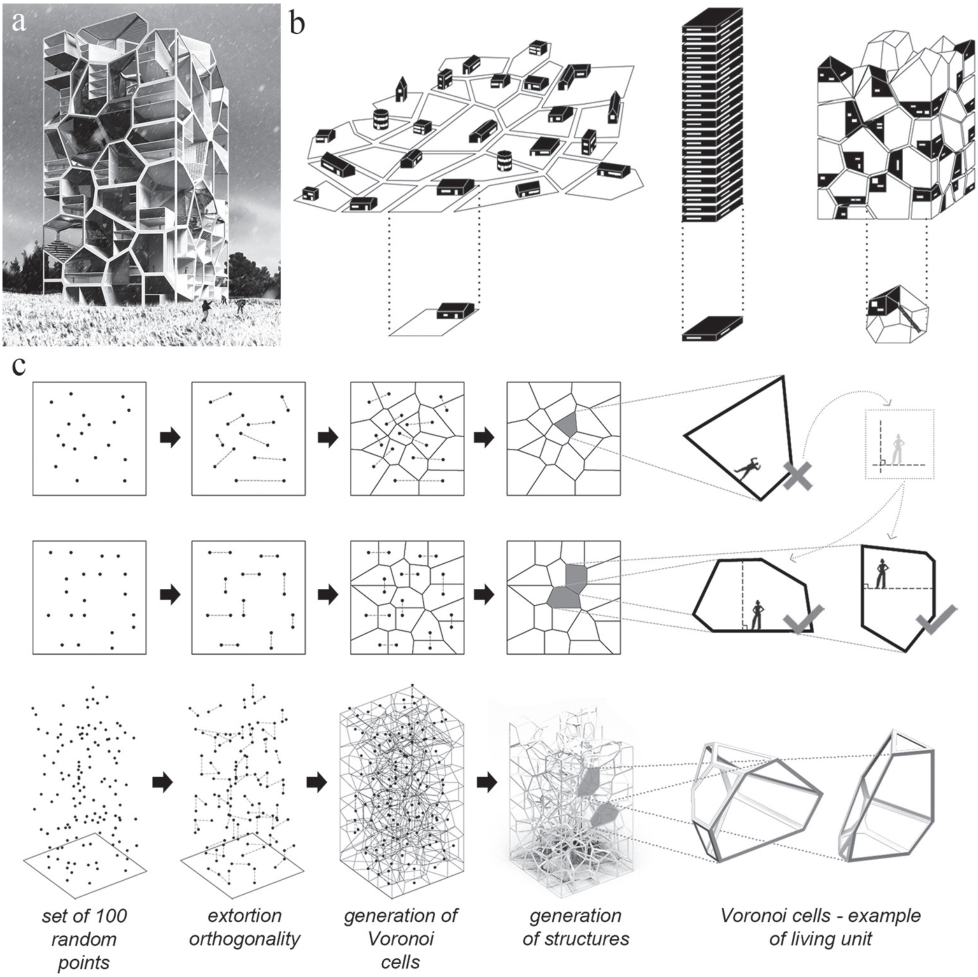


Fig. 3. Voronoi Diagram application in three-dimensional structural surface discretization; Vertical Village, Yushang Zhang, Rajiv Sewtahal, Riemer Postma and Qianqian Cai project, 2011; a – object visualization; b – block diagram; c – rules for generating Voronoi divisions into residential units

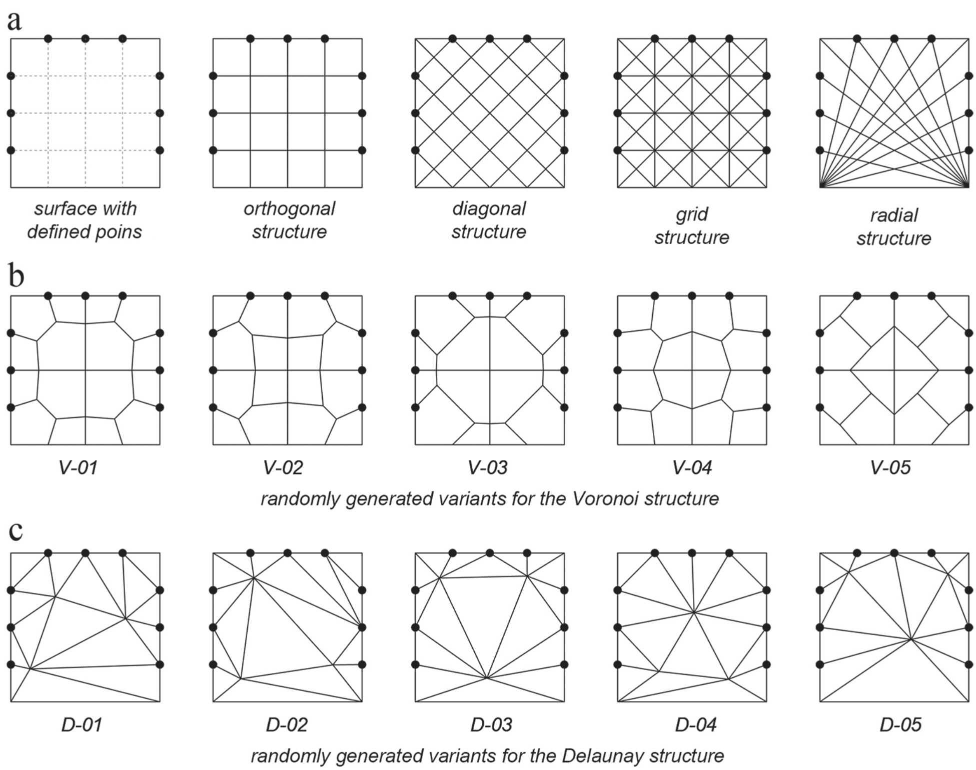


The roof structure was constructed of same-diameter steel pipes, but three different wall thicknesses. As a result of a joint implementation, rapid assembly was possible of the welded installation elements on the spot. The curved roof area was filled with transparent, pneumatic EFTE film cushions tailored to match the shape of individual cells.

The Voronoi diagrams can also be used in the discretization of curved surface [Wysokińska E., Rokicki W., 2013]. An example of such shape is the pavilion – meditation grotto, made as part of the research project by the Metalab Architecture & Fabrication group in cooperation with Gerald D. Hines, Mrs. Jane Blaffer Owen, professors Andrew Vrana, Joe Meppelink, Ben Nicholson and a group of students from the College of Architecture in Houston (Texas). The object was to function as a permanent landmark on the University of Houston campus. The project reflects the biomorphic processes, and its implementation was made possible with digital technologies used for 3D modeling, three-dimensional scanning and fabrication using CNC machines. The divisions were generated parametrically as minimal surfaces, inspired by the forms commonly found in marine organisms (Fig. 2b, c). The structural mesh was made of stainless steel (Fig. 2d).

The algorithm making up the Voronoi cell is used in three-dimensional architecture in order to build up structural forms. An example of this is the Vertical Village concept – a project of Yushang Zhang, Rajiv Sewtahal, Riemer Postma and Qianqian Cai architects (Fig. 3). The concept is based on the sustainable development principles that involve the use of the Voronoi Diagram to shape modern "house with a garden" residential units. The use of Voronoi Diagram geometry required the creation of an algorithm which would generate

Fig. 4. Examples of plane divisions for the designated points; a – typical, regular structures; b – Voronoi tessellation for specific boundary conditions generated using an algorithm; c – Delaunay triangulations for specific boundary conditions generated using an algorithm



functionally rational solutions. The result was a rectangular block consisting of Voronoi cells, each of which features a different "3D plot" proposal. By using only one method of space discretization, the whole concept is consistent as a composition and enables the development of the "Vertical Village".

Own Research for selected rod models

In the era of algorithmization tools for digital modeling, the creation of Delaunay rod systems is the most common process of generating multivariate solutions for specific boundary parameters [Rokicki W., Gawell E., 2013]. The selection of the optimal system becomes a daunting task, which requires, among others, the ability of the designer to rationalize technical solutions. The example in the illustration below (Fig. 4) presents selected plane divisions with designated 9 points (marked with black dots). For specific boundary assumptions, it is possible to determine typical, regular wire rod systems: orthogonal, diagonal, trussed or radial (Fig. 4). By selecting the same assumptions as above, you can generate numerous different cases for the Voronoi structure (Fig. 4b), or the Delaunay structure (Fig. 4c).

The bar systems formed by various methods of surface division were used to carry out own studies aimed at optimizing the structures of the rod due to the minimal material consumption criterion.The models mentioned above (Fig. 4) present a limited area for theoretical exploration and choosen research methodology could be also applied to the search for more complex structural systems.

For the analyzed flat rod systems, comparable boundary conditions have been defined i.e.: external outline (field size 9.0x9.0m), support scheme and the load system (Fig. 5). The buckling lengths of support elements have been adopted as in the case of the truss bars.

Fig. 5. The assumptions and results of own analytical research for flat bar-like structures; a – assumed static diagram; b – polynomial approximation chart for $p^3(xi)$ and yi value (mass) for each flat Voronoi and Delaunay structure

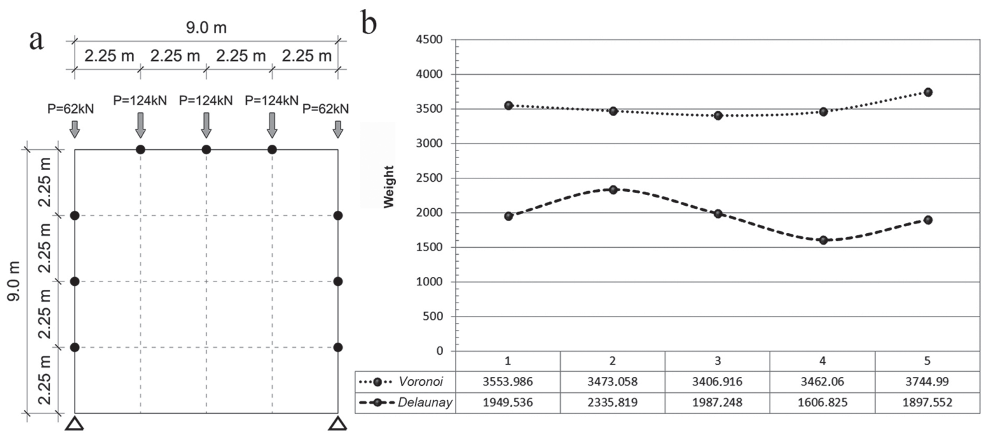


Table 1. Summary of results for a typical, regular structures

Type of structure	Number of joints	Profile /type, dimensions/[mm]	Total length of elements [m]	Total weight [kg]
Orthogonal	25	RK 200x200x6	90.00	3265
Diagonal	41	RK 140x140x5	137.76	2888
Trussed	41	RK 100x100x4	141.04	1683
Radial	56	RK 70x70x3	184.24	1149

The bar system is supported on the two articulated supports. The following fixed and variable loads have been adopted: the dead load and the ceiling reactions are treated as a load constituting a separate variant included into a load combination with a factor of 1.5, as calculated for a dynamic load. The structure load was applied to the upper truss chord. The load combination was in accordance with PN-EN 1990 standard. Structural steel S235 square cross-section profiles were chosen for the analysis. All bars of the structure shared the same cross-section. In order to obtain comparable results, the element database was limited to square cross-section pipes, type RK PN-EN 10210-2: 2000. The desired solution for each bar system in the analysis was the selection of cross-sections, which would be most efficient in the use of their load capacity.

The first group of bar systems researched were typical structures (Fig. 4), the most effective of which proved to be radiant (the upper chord forces are transferred directly to the support). The second and third groups consisted respectively of topological Voronoi and Delaunay systems (genus of: a fixed number of joints and planes), randomly generated in the Rhinoceros Grasshopper program. Despite the changing geometry of divisions both within Delaunay and Voronoi structures, the characteristics impact of topological transformations on the mass of

Table 2. Summary of results for randomly generated Voronoi structures

Voronoi structure variant	Number of joints	Profile /type, dimensions/[mm]	Total length of elements [m]	Total weight [kg]
V01	16	RK 220x220x6	88.04	3526
V02	16	RK 220x220x6	89.52	3585
V03	16	RK 200x200x6	89.28	3239
V04	16	RK 220x220x6	89.24	3574
V05	16	RK 220x220x6	92.80	3717

Table 3. Results for randomly generated Delaunay structures

Delaunay structure variant	Number of joints	Profile /type, dimensions/[mm]	Total length of elements [m]	Total weight [kg]
D01	16	RK 120x120x5	109.00	1943
D02	16	RK 140x140x5	112.66	2362
D03	16	RK 120x120x5	109.29	1948
D04	16	RK 100x100x5	111.22	1633
D05	16	RK 120x120x5	106.10	1891

individual systems is the variable flow curve (Table 2, Table 3 and Fig. 5b). This means that regardless of the topological transformations in the structure (and to some extent also the metric change), it is possible to describe the approximate effectiveness of the systems.

Summary

Today, we observe an increased interest in bionic trends and the digital tools supporting architectural design with the use of discretization methods of the surface simulate the laws governing processes in nature e.g. as fractal structure generators. The Voronoi tessellation, as well as the Delaunay triangulation which geometrically relates to it, both constitute bionic methods of dividing the surface that can be used in architecture in order to design a number of original structural elements for roofs, facades, walls, etc.

Thanks to the booming digital design optimization processes, there is an increase among designers in the awareness of the logical use of bionic structures, as well as the ability to use generative modeling methods to determine the criteria for a more detailed search. In the interdisciplinary approach to architectural design, the flexibility as the ability to adapt and transform, is an important feature of digital modeling. The ability to generate an infinite number of alternative solutions for given parameters is a considerable advantage of digital space discretization methods in the search of architectural and structural efficiency.

As a result of the analysis, it was observed that the bionic structures, both Voronoi and Delaunay, were each part of the optimal solutions, constituting rational bar systems due to the assumed minimum mass criterion. The possibility of shaping multivariate structures for the same parameters was an additional advantage of the bionic meshes. Due to the efficiency of their structure, Voronoi Tessellation may be an alternative to typical solutions in the search for rational and at the same time tectonically new bar systems.

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Diagramy Voronoi – modele badawcze struktur prętowych w optymalizacji architektoniczno-konstrukcyjnej

STRESZCZENIE

W architekturze, we współczesnym projektowaniu, obserwujemy interesującą tendencję, czyli kształtowanie bardzo skomplikowanych układów przestrzennych, inspirowanych wzorami zaczerpniętymi ze świata przyrody. W wyniku takich działań powstają rozwiązania bardzo nowatorskie wizualnie, ale także, co jest istotne z inżynierskiego punktu widzenia, efektywne technologicznie. Logika kształtowania „bionicznej" architektury jest ukierunkowywana m.in. na racjonalne zużyciu materiałów i energii. Poszukiwania przez wielokryterialne analizy nowych struktur przestrzennych powodują wzrost zainteresowania zagadnieniami cyfrowej optymalizacji w interdyscyplinarnych analizach architektoniczno-konstrukcyjnych. Szczególną rolę w doskonaleniu programów komputerowych zaczynają odgrywać algorytmy, umożliwiające kształtowanie różnorodnych, wielowariantowych układów nośnych w oparciu o podobne założenia brzegowe. Jako przykład takiego działania przedstawiono próby badawcze w kształtowaniu regularnych oraz nieregularnych struktur prętowych, w oparciu o opracowane własne modele cyfrowe. Przeprowadzone symulacje dotyczyły analizy konfigurowanych struktur z uwagi na ich efektywność w zużyciu materiału.

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